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THE "BURDEN OF THE DEBT" AND THE NATIONAL INCOME[1]

By EVSEY D. DOMAR*

I

"Full employment after the war" has now become the subject most frequently discussed by economists. When the war is over, the level of employment and income will be determined to a great extent by the speed and character of the reconversion process. After that, hopes of maintaining full employment are based, for good or for ill, on the various backlogs developed during the war. But when both periods are over, the old and so painfully familiar problem of the disposal of intended savings will again appear.

It is possible that private investment will be able to absorb all savings year in and year out, or that private investment will at least fluctuate around a sufficiently high average so that deficits which may be incurred by the government in some years will be offset by surpluses made in others. Whether or not this will actually happen is a matter of opinion; it is a problem not discussed here. Instead I propose to examine the less optimistic case, when private investment is insufficient to absorb intended savings over a relatively long period of time.

Public investment financed by borrowing, though perhaps the most direct and evident, is by no means the only method of dealing with the situation. The income-generating properties of various kinds of taxation still remain to be explored;[2] the possibilities of encouraging private investment by means of various tax devices have not been sufficiently worked out either; the same can be said about plans designed to reduce the propensity to save. It will be assumed here, however, either that all these measures cannot be tried, or that they have not proved sufficiently effective so that a continuous policy of deficit financing must still be pursued.[3]

* Mr. Domar is with the Board of Governors of the Federal Reserve System. The opinions expressed are those of the author and do not necessarily reflect the position of the Board.

[1] Thanks are due to Miss Mary Painter for her assistance in the preparation of this paper.

[2] See, however, P. A. Samuelson, "Full Employment After the War" in *Postwar Economic Problems* edited by S. E. Harris (New York, McGraw-Hill, 1943), p. 44; A. H. Hansen and H. S. Perloff, *State and Local Finance in the National Economy* (New York, Norton, 1944), pp. 245-46; L. A. Metzler, "Effects of Income Redistribution," *Rev. Econ. Stat.*, Vol. 25 (Feb., 1943), pp. 49-57; B. Ruml, *National Fiscal Policy and the Two Super Budgets,* an address delivered before the Institute of Public Affairs, University of Virginia, June 27, 1941.

[3] At this stage, "public investment financed by borrowing" and "deficit financing" are

The theory of the multiplier and our actual experience during this war have demonstrated, I believe, that money income can be raised to any desired level if the total volume of public expenditures is sufficiently high. This view will probably be accepted also by the opponents of deficit financing. Their objections to such a policy are based on several grounds, the most important being the belief that continuous government borrowing results in an ever-rising public debt, the servicing of which will require higher and higher taxes; and that the latter will eventually destroy our economy, or result in outright repudiation of the debt.

That continuous borrowing will result in an ever-growing public debt is evident; that, with a non-falling interest rate, the interest charges will grow is likewise true; and finally, assuming—as we shall in this paper—that all funds for payment of interest charges are to be raised by taxation,[4] there is no question that the absolute amount of taxes to be collected for that purpose will increase at the same rate. But all these *absolute* amounts do not mean much.

Whatever favorable or unfavorable effects the existence and growth of the debt may have, what matters is its relation to other economic variables, such as national income, resources of the banking system, volume of private securities outstanding, and so on, the particular relation to be studied depending on the character of the problem at hand. The phrase "burden of the debt," if it has any meaning, evidently refers to the tax rate (or rates) which must be imposed to finance the service charges, and that the *tax rate* will rise is far from evident.

The belief that government borrowing must necessarily result in rising tax rates is so widespread both in technical and popular writings that no quantitative analysis of it has, to my knowledge, ever been made. It has been pointed out, however, particularly by Professor Hansen, that the debt problem should be studied in its relation to national income, and that with a growing national income the "debt burden" is likely to be confined within manageable limits.[5] The proponents of deficit financing have also argued that the burden of a domestically-held debt depends to a great extent on the distribution

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used synonymously. The essential fact is that government absorbs the savings and spends them. The nature of these expenditures will be discussed in Section IV.

[4] This assumption is made both to simplify the argument and to protect the reader from a shock. To many, government investment financed by borrowing sounds so bad that the thought of borrowing to pay interest charges also is simply unbearable.

[5] A. H. Hansen and Guy Greer, "The Federal Debt and the Future," *Harpers Magazine*, Apr., 1942, pp. 489-500; A. H. Hansen, *Fiscal Policy and Business Cycles* (New York, Norton, 1941), pp. 135-185; "Moulton's *The New Philosophy of Public Debt*" in Hansen and Perloff, *op. cit.*, pp. 285-298; and his other writings.

of the debt ownership;[6] that however large the debt may be, interest charges can still be collected because interest income constitutes a part of taxable income;[7] and finally, that a tax rate, however high, will not deter investment if losses can be offset against other income.[8]

No evaluation of these last three arguments will be made here. But the issues of the debt problem will appear clearer if we adopt the attitude of the opponents of deficit financing and treat this tax rate as a burden, as a price for the privilege of having a higher level of income (and employment) than would prevail without deficit financing. We shall therefore explore the behavior of the tax rate over time under several sets of assumption. In addition, it will be interesting to examine what the community gets for this payment, *i.e.*, the net income of the non-bondholders after the transfer of interest charges to the bondholders has taken place.

It is true that the existence and growth of the debt raise a number of other problems besides the behavior of the tax rate and of the net income of the non-bondholders. I hope it will be recognized, however, that these two variables are the most important ones, and that an analysis of their behavior will be of considerable help in the understanding of the whole problem of the debt.

The paper is based on several dynamic models which are developed mathematically. All mathematics, however, is concentrated in the Mathematical Appendix and only the final results are given in the text. As in most investigations of this character, certain simplifying assumptions will have to be made, but ways of modifying them will become apparent as the argument progresses.

II

The burden of the debt, or the average tax rate covering the interest charges, equals, roughly speaking, the ratio of the interest charges to income; or the ratio of the debt to income multiplied by the interest rate paid on bonds.[9] *It will be assumed that this interest rate is a given constant* (i). If we now want to find the effects of

[6] A. H. Hansen: sources given in footnote 5; A. P. Lerner, "Functional Finance and the Federal Debt," *Social Research*, Vol. 10 (Feb., 1943), pp. 38-51; Stuart Chase, *Where's the Money Coming From?* (New York, Twentieth Century Fund, 1943), pp. 97-110.

[7] Lerner, *op. cit.*, S. E. Harris, "Postwar Public Debt" in *Postwar Economic Problems* edited by him, pp. 169-186. Unfortunately both Lerner and Harris assumed perfectly arbitrary magnitudes of the debt and income without any analysis of their interrelationship.

[8] Lerner, *op. cit.* For a more elaborate analysis of the effects of loss offset, see E. D. Domar and R. A. Musgrave, "Proportional Income Taxation and Risk Taking," *Quart. Jour. Econ.*, Vol. 58 (May 1944), pp. 388-422.

[9] Though not quite correct, this statement will do for the time being. A more correct one will be given on pp. 802-03.

deficit financing on the tax rate, we should examine its effects on the magnitude of the debt and of the national income.

The effect of borrowing on the debt is somewhat complex and will be taken up in Section III. At this stage we can only record the obvious fact that continuous borrowing will of course result in an ever-increasing debt. Indeed, this point has never been overlooked in the numerous writings on the subject.

The other relevant fact—that deficit financing may have some effect on income—has received a different treatment. Opponents of deficit financing often disregard it completely, or imply, without any proof, that income will not rise as fast as the debt. On the other hand, we sometimes get the incorrect impression that it is sufficient for the government to spend, say $100, and the national income will *rise* by $300 or $400, depending on the magnitude of the multiplier. If this were really so, there would be no debt problem at all: it would certainly pay us to *raise* the national income by $300 at the expense of some $2.00 increase in interest charges.[10]

A clear distinction should be made between *levels* of investment expenditures and income and *increments* in investment expenditures and income. With a given average propensity to save, the level of national income will be a multiple of the level of investment expenditures (public or private). Similarly, with a given marginal propensity to save, an increment in national income will be a multiple of an increment in investment expenditures. But neither of these two statements tells anything about the relation between the *level* of investment expenditures and an *increment* in income.

It should be emphasized that the stimulating effects of a given increment in expenditures tend to disappear quite soon, unless, of course, one believes in pump-priming which does not at present find many proponents. Pump-priming aside, an increase in national income of, say, $300 produced by an increase in investment expenditures of, say, $100 will presently disappear and income will fall back to its former level. But the public debt (if investment expenditures are financed by government borrowing) has permanently increased (by $100), and so have interest charges (by $2.00). This is the source of the debt problem. If the national income is to be maintained at the new level, new amounts must be spent.[11]

In order to simplify the problem, *it will be assumed that the community's average and marginal propensities to save are equal and*

[10] That is, 2 per cent of the $100 borrowed.

[11] That this is so can be easily demonstrated by means of algebra, a numerical table or a chart. For a good example, see Hansen, *Fiscal Policy and Business Cycles,* Chart 16, p. 272. It was from this chart that the present paper originated.

constant.[12] Under this assumption, national income will be simply a multiple of investment expenditures, and the two series will behave in exactly the same manner.[13] To maintain a *constant* level of income it is sufficient to have a *constant* stream of investment expenditures, public and private, but to achieve a *rising* income, total investment expenditures must also be *rising*. Thus, if it is desired that income should rise at a constant absolute rate, total investment expenditures must also rise at a constant absolute rate; or if income is to rise at a constant percentage rate, investment expenditures must also rise at a constant percentage rate; and so on. In other words, by regulating the total investment expenditures, national income can be made to behave in any desired manner.

All this refers to *money* income. Nothing has been said so far about *real* income. Whether or not real income will follow the movements of money income depends on a number of circumstances which will be discussed briefly in Section IV. But it will greatly simplify our analysis if *we now assume that the price level remains constant* (whatever that means over long periods of time), *so that changes in money income and in real income are the same*.[14]

Before proceeding to the actual analysis of our problem, two other questions have to be settled. The first refers to the distinction between national income and taxable income. Without getting into current controversies, it will be sufficient to define *national income* as the sum of all wages, salaries, dividends, etc., paid out plus undistributed corporate profits, but excluding interest paid on the public debt. *Taxable income* will be defined as the national income *plus* interest receipts on the public debt, since interest receipts are also subject to taxation. It will be assumed that service charges are raised by means of a proportional income tax imposed on the total taxable income (without any exemptions), so that the tax rate will equal the ratio of interest

[12] This would be a bad assumption to make in any problem of cyclical character. It may be quite reasonable, however, in an analysis of a secular problem such as ours. More about it will be said in Section IV, pp. 821-22.

[13] This of course follows from the definition of the propensity to save. Using I for investment, Y for income and λ for propensity to save, we have $Y = I \cdot \frac{1}{\lambda}$ so that if $I = f(t)$ where t is time, $Y = f(t) \cdot \frac{1}{\lambda}$.

[14] It is well to recognize that the assumption of a constant price level considerably reduces the quality of the analysis. As a matter of fact, in three out of the four cases to be analyzed (1, 2 and 4), a constant price level is quite unlikely to be maintained. But the purpose of this paper is to study the debt problem in its bearing on deficit financing. It, therefore, appears worth while to sacrifice some theoretical completeness in order to bring out clearly the essence of the problem. I do not think that the validity of the final conclusions is thereby impaired.

charges to taxable income, it being understood that taxes levied for other purposes than to service the debt have already been subtracted in arriving at this definition of national income.[15]

Since no mathematical derivations are given in the text, it will be necessary to construct numerical tables to demonstrate the argument. It must be made perfectly clear that these tables are given as an illustration only and do not represent any attempt to forecast. They cover a period of 300 years not because I expect deficit financing, in the accepted sense of the terms, to last that long, but simply to convey the notion of a long period of time.

To construct the tables, the parameters used must be given numerical values. An effort to take reasonable magnitudes could as readily be made.

Let the debt at the beginning of the "experiment" = \$300 billion, the national income at the beginning of the "experiment" = \$130 billion, the interest rate on the debt, i, = 2 per cent.

In addition, a decision must be made with regard to the magnitude of government borrowing. To do this, we must have some idea about the community's propensity to save. The examination of Professor Kuznet's estimates shows that over the period 1879-1928 net capital formation constituted about 13 per cent of national income (in 1929 prices). This percentage appears to have been remarkably stable, with a slight downward trend; in the decade 1919-28 it was about 10.6 per cent.[16] There may be serious objections against this kind of approach to an estimate of a future secular propensity to save under conditions of full employment, but it is a question which cannot be discussed here. I shall assume that the propensity to save will be 12 per cent. How this 12 per cent will be divided between private and public investment is again a matter of guesswork. It can just as well be assumed that they share in it equally. In other words, the fraction of national income borrowed by the government, to be indicated by α, will be assumed to equal 6 per cent.[17]

[15] Disposable income after taxes will equal taxable income minus tax collections, *i.e.*, national income, since interest charges equal tax collections. It appears reasonable to apply the propensity to save to *disposable* income, and the fact that it equals national income considerably simplifies the mathematics of the problem.

[16] It may be well argued that non-deflated series should be used. Numerically, the difference is very small, and there is no need to elaborate this point any further here. Source: Simon Kuznets, an unpublished revision of Table 2 in *Uses of National Income in Peace and War,* Occasional Paper 6, March 1942, National Bureau of Economic Research (New York, 1942), p. 31.

[17] Some remarks about a rising propensity to save and a rising α will be made in Section IV, pp. 821-22. In addition, a variable percentage of national income borrowed by the government is discussed in Case 4 (The War Model), pp. 812-16.

By referring to the Mathematical Appendix, the reader can easily construct other tables based on different numerical magnitudes of the parameters.

III

All preliminaries having been disposed of, a direct attack on the problem can now be made, which is to find out what the tax rate and other variables will be when national income is made to behave in a given manner.[18] Theoretically, there is an infinite number of patterns which the national income may be assumed to follow, but only the simplest ones will be considered here. It is clear that, in a problem of this type, it is more meaningful to express the growth of income in percentage rather than absolute terms, and a function with a constant percentage rate of growth will occupy the center of the discussion (Case 3).[19] But it may be also interesting to examine the situations when income is held constant (Case 1), or is increasing at a constant absolute rate (Case 2). Finally, a variable percentage of income borrowed by the government is analyzed in the so-called "War Model" (Case 4).

Case 1. When National Income Remains Constant

Since the government keeps borrowing α per cent of national income, it is evident that the debt will increase at a constant absolute rate. The ratio of the debt to national income will therefore grow without limit and the tax rate will approach asymptotically 100 per cent.[20] The net income after taxes of the non-bondholders will approach zero. The picture is rather dismal.

Actually, it takes quite a long time before conditions become really bad, depending of course on the magnitude of the parameters. As shown in Table I, after 50 years the tax rate is only about 10 per cent, and it takes almost 250 years to bring it to 25 per cent. But there is something inherently odd about an economy with a continuous stream of investment expenditures and a stationary national income. There may exist at least two explanations:

(1) Investment expenditures do not result in a higher per manhour

[18] As stated on pp. 801-02, national income is made to behave in a given manner by regulating the volume of investment expenditures. Investment expenditures are the independent variable. This must be borne in mind, because the discussion in this section might give the misleading impression that national income is the independent variable.

[19] From a realistic point of view, a function with a slowly declining percentage rate of growth would probably be more significant. This paper being but a first step in an analysis of this type, I thought it better to make no use of the more complex functions. A declining percentage rate of growth is, however, discussed in Section IV.

[20] It may appear strange that the tax rate does not go beyond 100 per cent, in view of the fact that the ratio of the debt to income increases without limit. But the tax rate is the ratio of the interest charges to the *taxable* income, and as the debt and therefore the interest charges grow, taxable income increases as well. It is on this fact that Harris and Lerner based their defence of a large public debt, as already mentioned in footnote 7.

productivity, and there is no increase in the number of manhours worked. It is doubtful whether these expenditures should be called *investment* in the first place. But such a situation is not incompatible with full employment, if the level at which national income is kept is sufficiently high.

TABLE I.—THE TAX RATE AND THE RATIO OF THE DEBT TO NATIONAL INCOME WHEN NATIONAL INCOME REMAINS CONSTANT

Original debt =$300 billion
Original income=$130 billion
α=6 per cent
i=2 per cent

Years	Tax Rate Per Cent	Ratio of Debt to National Income
0	4.41	2.31
1	4.52	2.37
2	4.63	2.43
3	4.74	2.49
4	4.85	2.55
5	4.96	2.61
10	5.50	2.91
15	6.03	3.21
20	6.56	3.51
25	7.08	3.81
30	7.60	4.11
40	8.61	4.71
50	9.60	5.31
75	11.98	6.81
100	14.25	8.31
125	16.40	9.81
150	18.44	11.31
175	20.40	12.81
200	22.25	14.31
225	24.02	15.81
250	25.71	17.31
275	27.33	18.81
300	28.88	20.31
At the limit	100.00	Infinitely large

(2) As a result of the investment expenditures, productivity per manhour rises, but there is a continuously falling number of manhours worked. It may mean an ever shortening work-week. Under present institutional conditions, it is more likely to mean ever increasing unemployment. Together with the ever rising tax rate, it would combine the bleakest prophesies of both Karl Marx and the *Wall Street Journal*.[21]

[21] There is, of course, a third possibility, namely, that of a falling price level, so that the real income would be actually rising. Such a case would exclude neither increasing productivity nor full employment. It is worth further study. What really matters is the fact that an ever increasing share of the national income goes to the bondholders. This of course

To repeat, continuous government borrowing not accompanied by a rising national income results in an ever, though slowly, rising debt burden in addition to other possible economic dislocations already mentioned. How long such a policy can be pursued is a matter of conjecture. It will be shown in Cases 2 and 3, however, that the difficulty lies not in deficit financing as such, but in its failure to raise the national income. To have a rising income, investment expenditures (public and private) must not remain constant, but must increase.

Case 2. When National Income Increases at a Constant Absolute Rate

As the percentage of income borrowed (α) is constant, by assumption, and the income grows at a constant absolute rate, the annual deficits become larger and larger, so that the debt itself grows at an accelerated absolute rate.[22] Therefore the ratio of the debt to national income will rise without limit, and the tax rate will again approach 100 per cent.

It is of course evident that in the present case the absolute magnitude of the income is larger than it was in Case I. It is equally evident that a more rapidly growing income will, with our assumptions, result in a larger debt. We might therefore expect that the tax rate (and the ratio of the debt to income) will be the greater the more rapidly income rises. Actually, exactly the opposite holds true.

Table II compares the tax rates resulting from a constant income (as in Case I) and from income rising at 5 and 10 billion dollars per year, respectively. After 50 years, the tax rate equals 9.6 per cent when income is constant, 5.3 per cent when it rises at 5 billions per year, and only 4.4 per cent when the rate of growth equals 10 billions. It takes about 280 years to raise the tax rate to 15 per cent when income increases at 10 billions per year, and only 110 years when it remains constant. And in general, it can be easily shown,[23] that *the faster income rises the lower will be the tax rate,* even though a more rapidly rising income results in a larger absolute magnitude of the debt. This point will be taken up again in Case 3 and in Section IV.

It is still true, however, that we are confronted with an ever rising tax rate. It could therefore be expected that the net income after taxes of the non-bondholders would gradually approach zero as it did in

raises grave doubts as to the advisability of fiscal and price policies resulting in a constant money and a rising real national income.

[22] Mathematically speaking, this means that while national income is linear, the debt, being a function of the integral of income, is a quadratic. See Mathematical Appendix, p. 824.

[23] See Mathematical Appendix, p. 824.

Case 1. But this growth of the tax rate is more than offset by the ever rising national income, so that the net income of the non-bondholders after taxes approaches a very high asymptote.[24] It therefore follows that the non-bondholders will be much better off than they were at the beginning of the experiment, in spite of the rising tax rate.

TABLE II.—A COMPARISON OF TAX RATES WHEN NATIONAL INCOME REMAINS CONSTANT AND INCREASES AT $5 BILLION AND $10 BILLION PER YEAR (IN PERCENTAGES)

Original debt = $300 billion — α = 6 per cent
Original income = $130 billion — i = 2 per cent

Years	Constant Income	Income Increasing at $5 Billion per Year	Income Increasing at $10 Billion per Year
0	4.41	4.41	4.41
1	4.52	4.36	4.22
2	4.63	4.32	4.06
3	4.74	4.29	3.92
4	4.85	4.26	3.80
5	4.96	4.24	3.71
10	5.50	4.18	3.43
15	6.03	4.22	3.35
20	6.56	4.29	3.37
25	7.08	4.42	3.47
30	7.60	4.56	3.61
40	8.61	4.91	3.96
50	9.60	5.31	4.37
75	11.98	6.41	5.52
100	14.25	7.57	6.74
125	16.40	8.75	7.95
150	18.44	9.92	9.16
175	20.40	11.08	10.35
200	22.25	12.21	11.54
225	24.02	13.33	12.33
250	25.71	14.42	13.77
275	27.33	15.49	14.86
300	28.88	16.53	15.92
At the limit	100.00	100.00	100.00

But it is doubtful, nevertheless, whether an economy with an ever rising tax rate levied for the sole purpose of paying interest on the debt will be able to escape serious economic and social difficulties which may possibly lead to a repudiation of the debt.

What is the nature of the economy described in this model? We see

[24] This asymptote is given by the expression $\frac{2b}{\alpha i}$ where b is the absolute rate of increase of the national income, and i is the interest rate paid on the debt.

that larger and larger absolute amounts are invested (publicly and privately), but in spite of this, national income rises only by the *same* amount. The explanation of this phenomenon is practically the same as in Case 1:

(1) Investment fails to raise productivity per manhour sufficiently to allow the national income to grow faster; neither is there a sufficient rise in the number of manhours worked. In other words, the result is a diminishing productivity of investment which may be due to the wasteful character of investment expenditures, or to a lack of new technological improvements.[25]

(2) Productivity per manhour rises sufficiently, but there is a continuous decline in the number of manhours worked. This may mean more voluntary leisure or more unemployment.

If it is unemployment that prevents national income from rising faster (*e.g.*, at a constant percentage rate), the remedy is simple (at least in theory): investment expenditures should proceed at a faster rate. But if productivity per manhour fails to advance sufficiently, the situation is more serious. This question will be taken up in Section IV.

Case 3. When National Income Increases at a Constant Percentage Rate

Since Case 3 is the most important model, the major part of the subsequent discussion refers to it. Use will be made here of three symbols, two of which have already been introduced:

α—percentage of national income borrowed,

i—interest rate paid on bonds,

and

r—percentage rate at which national income increases.

To understand the relationship between the debt and income in this case, it is necessary to make use of the following two propositions *on which the whole analysis rests*:

1. If a variable Q is the sum of $q_1, q_2, q_3, q_4, \ldots$ and so on, each of which is r per cent larger than the preceding one, then the addition of more and more q's makes Q itself increase at a rate approaching r per cent.

2. If any two variables increase at the same percentage rate, the ratio between them remains constant.

[25] Productivity of investment as used in this paper refers to an increment in *national income* due to a given investment, and not to return over cost received or expected by an investor, which forms the essence of Keynes's marginal efficiency of capital and allied concepts.

Mathematically, both propositions can be proved very simply.[26] The non-mathematical reader can construct numerical tables and plot the results on semi-logarithmic paper. He will find that as time goes on, his sum, whose components grow at a constant percentage rate, will look more and more like a straight line, *i.e.*, its rate of growth will approach a constant. If he plots two functions growing at the some constant rate, they will be represented by two *parallel* straight lines.

Now, according to our assumption national income grows at a constant percentage rate r. Since every year a constant α percentage of that income is being borrowed, it is clear that the deficits also grow at r per cent per year. The total debt is simply the sum of all the deficits. Therefore, according to the first proposition, the rate of growth of the debt itself will also approach r, and according to the second proposition, *the ratio between the debt and the national income will approach a constant.* This conclusion presents a striking contrast with the results obtained in Cases 1 and 2 where the ratio of the debt to income increased without limit.

It is shown in the Mathematical Appendix (pp. 824-25) that the

[26] *The first proposition:*

A proof not involving the use of calculus: as stated in the text, let

$$Q = a + a(1+r) + a(1+r)^2 + \ldots\ldots\ldots a(1+r)^t$$

where a is the original value of Q, r is the percentage rate of increase, and t indicates the number of years. We have here a geometric progression in which $(1+r)$ is the common ratio. Its sum is

$$Q = \frac{a[(1+r)^{t+1} - 1]}{r}$$

As t increases, Q approaches the expression

$$\frac{a}{r}(1+r)^{t+1}$$

which increases at r per cent per year.

The reader familiar with calculus can use a continuous function. If

$$\frac{dQ}{dt} = ae^{rt}$$

over the interval from 0 to t, then

$$Q = a\int_0^t e^{rt}dt = \frac{a}{r}(e^{rt} - 1),$$

which increases at a rate approaching r as t becomes large.

The second proposition:

Any two variables increasing at the same rate r can be expressed as $a_1(1+r)^t$ and $a_2(1+r)^t$ (or a_1e^{rt} and a_2e^{rt}),

where a_1 and a_2 are constants. Their ratio equals $\frac{a_1}{a_2}$ which is also constant.

Gustav Cassel applied these principles to the relationship between capital and income. See his *On Quantitative Thinking in Economics* (Oxford, Clarendon Press, 1935), p. 24.

constant which the ratio of the debt to income approaches equals the simple expression

$$(1) \qquad \frac{\alpha}{r}.$$

Similarly, the average tax rate approaches the limit expressed by

$$(2) \qquad \frac{i}{\frac{r}{\alpha}+i}.$$

To obtain some idea of the magnitudes of these two expressions, numerical values must be given to r. We shall experiment with $r=2$ per cent and $r=3$ per cent.[27]

The ratio of the debt to national income will approach 3 when $r=2$ per cent, and 2 when $r=3$ per cent. The tax rate will approach 5.7 per cent and 3.9 per cent with $r=2$ and 3 per cent respectively. These figures and the examination of expressions (1) and (2) again show that *the greater is the rate of growth of income, the lower will be the*

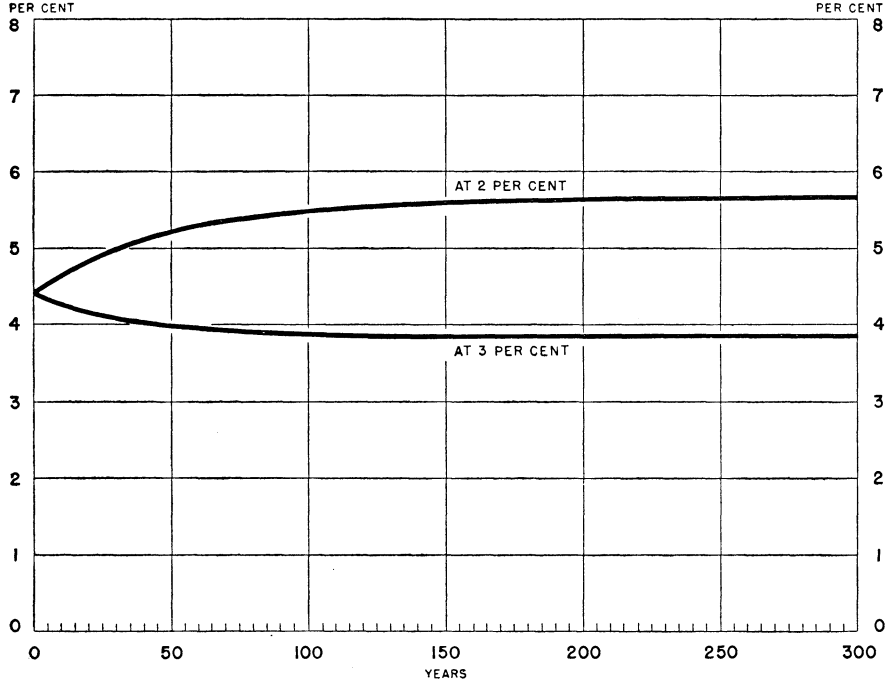

FIG. 1.—The Behavior of the Tax Rate When National Income Increases at a Constant Percentage Rate.

[27] A brief discussion of what r was in the past and may be expected to be in the future is presented in Section IV, pp. 817-20 and in Appendix B.

tax rate, even though a more rapidly rising income results in a larger absolute magnitude of the debt.

The net income of the non-bondholders after taxes will also grow at a rate approaching r per cent.

We thus see that, in spite of continuous government borrowing, the tax rate does not rise indefinitely but approaches a fairly reasonable limit. Even if private (net) investment disappears altogether, and the government has to borrow all the 12 per cent of income that the community desires to save, the tax rate will approach only 10.7 per cent and 7.4 per cent with r equal to 2 per cent and 3 per cent respectively.

TABLE III.—THE BEHAVIOR OF THE TAX RATE WHEN NATIONAL INCOME INCREASES AT A CONSTANT PERCENTAGE RATE (IN PERCENTAGES)

Original debt = $300 billion — α = 6 per cent
Original income = $130 billion — i = 2 per cent

Years	r = 2 per cent	r = 3 per cent
0	4.41	4.41
1	4.44	4.40
2	4.46	4.38
3	4.49	4.36
4	4.51	4.35
5	4.53	4.33
10	4.64	4.27
15	4.74	4.21
20	4.82	4.16
25	4.91	4.11
30	4.98	4.08
40	5.10	4.02
50	5.21	3.97
75	5.39	3.91
100	5.49	3.87
125	5.56	3.86
150	5.60	3.85
175	5.62	3.85
200	5.64	3.85
225	5.65	3.85
250	5.65	3.85
275	5.66	3.85
300	5.66	3.85
At the limit	5.71	3.85

Table III and Figure 1 show the behavior of the tax rate over time with $r = 2$ and 3 per cent. It is interesting to note that when $r = 2$ per cent, the tax rate approaches its asymptote from below up; while with $r = 3$ per cent, the corresponding asymptote is reached by a downward movement.[28] This latter situation takes place because the ratio

[28] In general, the movement will be up or down depending on whether the original magnitude of the debt is smaller or larger than $Y \cdot \frac{\alpha}{r}$.

of the debt to income 300/130 = 2.3 assumed here to exist at the beginning of the experiment is larger than the final ratio which equals 2; some doubt is, therefore, thrown on the soundness of the assumption that α will equal only 6 per cent. Evidently, greater percentages of national income were borrowed in the past, especially in periods of war.[29] It is of course hoped that the future will be free of wars. Still, it may be interesting to inquire what will happen to the variables if wars or other similar emergencies occur. This brings us to Case 4.

Case 4. The War Model

The amount of guesswork involved in the preceding three cases will appear negligible compared with the degree of imagination required from here on. Probably the best thing to do is to present a very dark picture and then find relief in the thought that the future will not be as bad as that.

Accordingly, let us assume that the future will consist of alternating periods of 25 years of peace (p) and 5 years of war (w); let the percentages of income borrowed be 6 (α) in peacetime, and 50 (β) during the war; and finally let the national income continue to grow at 2 (r) per cent per year.[30]

It can be easily shown by means of a table or a semi-logarithmic chart that the debt will grow very fast during wartime and more slowly in peacetime, but that *its average rate of growth will still approach r. Therefore the average tax rate will again approach a constant.*[31]

[29] Strictly speaking this means that the ratio of the debt to income $\frac{300}{130}$ is inconsistent with the assumed magnitude of $\frac{\alpha}{r} = \frac{6 \text{ per cent}}{3 \text{ per cent}} = 2$. If we retain the $\frac{300}{130}$ ratio, we should change α, r or both. As will be shown in Section IV and Appendix B, 3 per cent is a reasonable estimate of the rate of growth of the (real) national income in the past. Therefore the magnitude of α should be raised.

[30] This statement represents a drastic simplification of the problem. In particular, objections can be raised against our assumption of a constant price level, which is unlikely to prevail during these alternating periods of war and peace. During the wars, money income will probably rise much faster than at the rate of 2 per cent per year. But we can treat the 2 per cent rate as representing a long-run trend, to which the parameters apply. A comparison of methods of financing the last and the present wars (both in this country and in Great Britain) would indicate a movement toward a smaller reliance on borrowing; hence, the 50 per cent of income assumed to be borrowed during future wars is probably too high. If, however, this percentage is applied to the trend rather than to the actual money income, it will appear more reasonable.

The reader may also wonder whether an economy engaged in such frequent wars can expect to have a steadily rising income. This remains an interesting question.

[31] This statement will become clearer if we assume that the government borrows β (*i.e.*, 50) per cent of the national income *every* year. Then the tax rate, as given by (2) p. 810, will approach $\frac{i}{\frac{r}{\beta}+i}$ which is a constant. Since the actual percentage of income

Actually the behavior of the tax rate is more complex. As shown in Table IV and Figure 2, it fluctuates between two curves, reaching a maximum at the end of each war period and then going down to its

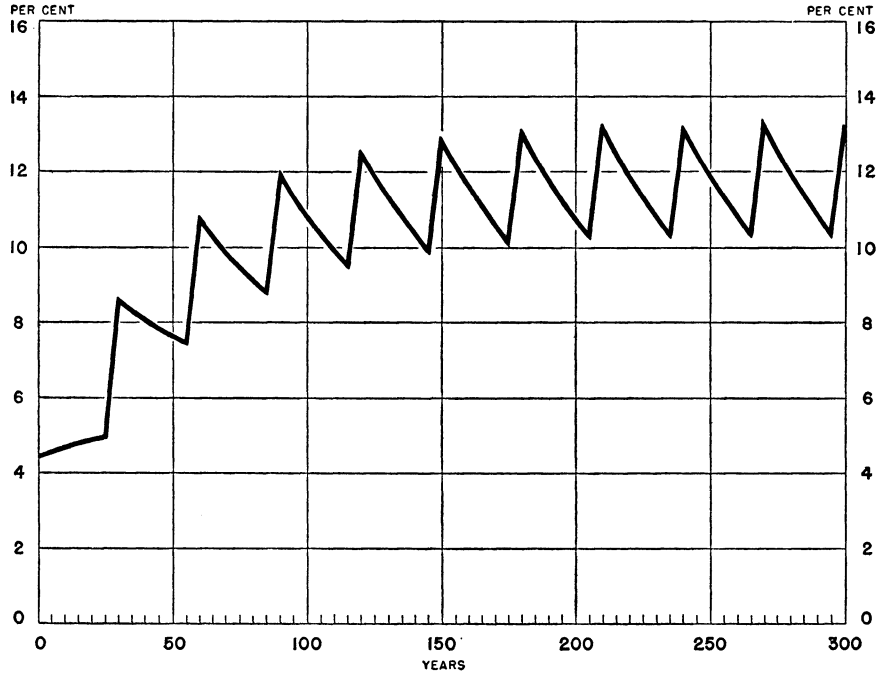

FIG. 2.—The Behavior of the Tax Rate in the War Model.

minimum at the end of each peace period. With the parameters used, the limits of these maxima and minima are:

Maximum	13.25 per cent
Minimum	10.42 per cent[32]

The ratio of the debt to national income will fluctuate in a similar manner, its maximum and minimum values approaching 7.64 and 5.82.

Since the tax rate and the ratio of the debt to income continue to fluctuate between their maximum and minimum values, it may be interesting to inquire what limits their *average* magnitudes approach.[33] The latter are given by the expressions:

---

borrowed is smaller than β the ratio of the debt to income must be below $\dfrac{i}{\frac{r}{\beta}+i}$.

[32] The formulas for these expressions are too complex to be reproduced here. See Mathematical Appendix, p. 825.

[33] I refer to simple arithmetic averages of actual tax rates (and ratios of debt to income) over the whole period of time.

(3) Average ratio of debt to income $= \frac{\sigma}{r} = 6.67$;

(4) Average tax rate $= \frac{i}{\frac{r}{\sigma} + i} = 11.76$ per cent;[34]

where

(5) $\sigma = \frac{\alpha p + \beta w}{p + w} = \frac{.06 \times 25 + .50 \times 5}{25 + 5} = 13.33$ per cent,

i.e., σ is the weighted average of percentages of income borrowed.

TABLE IV.—THE BEHAVIOR OF THE TAX RATE IN THE WAR MODEL

Original debt = $300 billion — r = 2 per cent
Original income = $130 billion — i = 2 per cent

Years	Tax rate Per Cent
0	4.41
1 peace time	4.44
2 peace time	4.46
3 peace time	4.48
4 peace time	4.51
5 peace time	4.53
25 end of peace	4.91
30 end of war	8.61
55 end of peace	7.48
60 end of war	10.77
85 end of peace	8.83
90 end of war	11.91
115 end of peace	9.55
120 end of war	12.52
145 end of peace	9.94
150 end of war	12.85
175 end of peace	10.16
180 end of war	13.04
205 end of peace	10.28
210 end of war	13.13
235 end of peace	10.34
240 end of war	13.19
265 end of peace	10.37
270 end of war	13.22
295 end of peace	10.39
300 end of war	13.24
At the limit	
end of war	13.25
end of peace	10.42
average	11.76

It is evident that the expressions (3) and (4) are identical to (1) and (2) (see page 810), respectively, except that α is replaced by σ.

[34] For a minor qualification of this formula see Mathematical Appendix, p. 825.

This fact makes the results obtained in Case 3 much more general. *It is no longer necessary that a constant percentage of income be borrowed every year. Variable percentages can be borrowed, and the α of Case 3 can then be treated as their weighted average.*

Whether the average tax rate of 11.8 per cent can still be regarded as "reasonable" is a matter of opinion. Those who expect it to ruin the economy should remember that more than half of it is due to government borrowing to finance the wars; as shown in Case 3, peacetime deficit financing resulted in a tax rate of only 5.8 per cent. But it is a curious fact that those who have been most vociferous against gov-

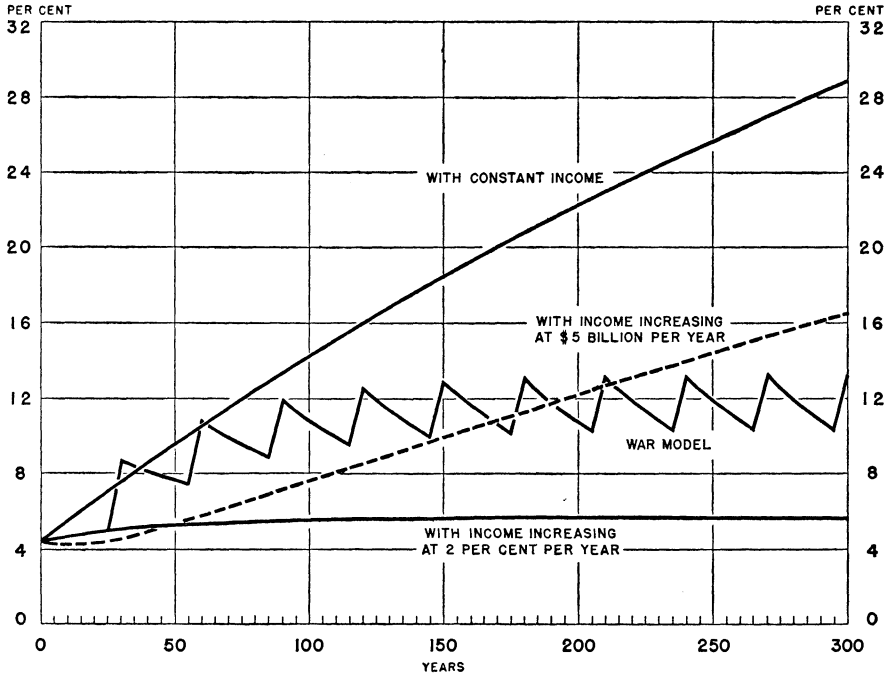

FIG. 3.—A Comparison of Tax Rates in Models 1-4.

ernment borrowing to achieve a high level of income and employment in peacetime have also opposed higher taxes during the present war!

Figure 2 has important implications for post-war fiscal policy. To repeat, the tax rate reaches its maximum at the end of the war, and then gradually declines during the peace period, *in spite of the fact that the government does not stop borrowing and the debt itself continues to rise.*[35] Now, some economic and political circles are burning with a desire to reduce the debt burden after the war. They recognize no other method of achieving their goal but by reducing the absolute size

[35] It is true, however, that the percentage of income borrowed does fall after the end of the war.

of the debt; that the government must stop borrowing is of course taken for granted. They should beware, however, lest the policies they advocate exert such a depressing effect on the national income as to result in an actually heavier debt burden, even though they succeed in paying off a part of the debt.

Finally, it may be worth while to compare the several tax rates obtained from the four cases discussed. In Case 2 it is assumed that income rises at 5 billion dollars per year; in Cases 3 and 4, at 2 per cent. Such a comparison is presented in Figure 3. It reveals the interesting fact that a constant percentage rate of growth of income is such a powerful force that we could engage in a 5-year war every thirty years and eventually come out with a lower tax rate than would be the case in continuous peace, but with the national income rising at a constant *absolute* rate!

IV

In Cases 3 and 4 of the preceding section, we have established that when national income grows at r per cent per year, the result at the limit is

$$\text{Ratio of debt to income} = \frac{\alpha}{r}, \tag{6}$$

and

$$\text{Tax rate} = \frac{i}{\frac{r}{\alpha} + i}, \tag{7}$$

where α can be interpreted either as a constant percentage of national income borrowed, or as a weighted average of variable percentages actually borrowed. As expression (7) for the tax rate looks rather complicated, it will be convenient—for purposes of exposition—to use an approximation to it, according to which

$$\text{Tax rate} = \frac{\alpha}{r}\, i.^{36} \tag{8}$$

The reader is reminded that a constant price level is assumed as before, so that movements of money income and real income are identical.

Expression (8) clearly shows that the burden of the debt is directly proportional to α and i and inversely to r. If the burden is to be light (with given α and i), there must be a rapidly rising income. *The prob-*

[36] This expression is derived from (7) by omitting i from the denominator, since i is apt to be quite small relative to $\frac{r}{\alpha}$. By this simplification, we are in fact assuming that interest on the debt is exempt from taxation. But numerically speaking, the mistake thus made is quite small and will be more than compensated for by convenience in exposition.

lem of the debt burden is a problem of an expanding national income.

How can a rapidly rising income be achieved?

If this question were asked in the pre-Keynesian era, the answer would be given in terms of manhours worked, productivity, and other *real* factors. Since the appearance of the *General Theory,* analysis has run in terms of investment expenditures, the multiplier, and other *monetary* considerations. Actually, there is no conflict in these two approaches: they simply state two sides of the same problem.

The real productive powers of economy establish the ceiling beyond which real national income, at any given time, cannot go, but whether or not it will reach this ceiling depends on the volume of expenditures actually made. If a rising income is desired, there must be both rising expenditures and rising productive capacity.

As explained in Section II, national income will grow at a constant percentage rate if and only if investment expenditures grow at the same rate (provided, of course, that the propensity to save remains constant). Since a stated fraction of these expenditures is made by the government out of borrowed funds, it follows that deficits must also grow at the same percentage rate. In absolute terms, the deficits must grow at an accelerated rate. It is horrifying to many to watch the public debt grow at an accelerated rate;[37] such a growth, however, is the only one which (with constant α and i) will *not* result in a rising burden of the debt.

From now on the heroic assumption is made that the stream of monetary expenditures will always be sufficient to maintain the national income at the maximum level established by the productive forces of the country. The growth of income will then be determined by the growth of these productive forces. Their behavior in the past and their expected rate of growth in the future represent an important and interesting subject which can be but briefly touched upon here. As a matter of fact, available past estimates refer to actually realized real income, and it can hardly be asserted that productive resources were always fully utilized even before the collapse of 1929.

Appendix B presents rates of growth of real national income for several countries, but the data are so fragmentary that not much reliance can be placed on them. For the United States, there are, fortunately, Professor Kuznets's estimates going back to 1879, which are presented in Table V. Over the whole period 1879-1928, total and per capita income grew at 3.3 and 1.5 per cent per year, respectively.[38]

[37] "Government spending tends to be like a drug, in that it takes larger and larger doses to get results, and all the time debt and taxes get higher and higher." National City Bank, *Economic Conditions* (Jan., 1944), p. 11.

[38] In regard to *money* income over the period 1879-1928, Professor Kuznets's estimates place the rates of growth of total and per capita income at 5.0 and 3.2 per cent, respectively. A comparison of these rates with the 3.3 and 1.5 per cent at which total and per

It is hard to form a definite opinion about their secular trend, because up to 1919 the estimates are presented only by (overlapping) decades, and the comparison between 1919 and 1929 is not very meaningful in view of the difficulty of measuring real output in a year like 1919. The general impression one gets from these figures is that there may have been some slackening of the rate of growth of total income, and

TABLE V.—PERCENTAGE RATES OF GROWTH OF REAL NATIONAL INCOME IN THE UNITED STATES, 1879–1929 (1929 PRICES)[a]

Period	Total	Per Capita
Annual averages by decades[b]		
1884–1894	2.8	0.7
1894–1909	4.2	2.4
1909–1914	3.1	1.5
1884–1914	3.6	1.7
1914–1919	1.8	0.4
1919–1924	2.9	1.5
1914–1924	2.4	0.9
1884–1924	3.3	1.5
Annual estimates		
1919–1923	5.4	3.7
1923–1929	3.5	2.1
1919–1929	4.2	2.7

Source: Simon Kuznets, an unpublished revision of Table 2 in *Uses of National Income in Peace and War*, Occasional Paper 6, March 1942 (New York, Nat. Bur. of Econ. Research, 1942), p. 31; and *National Income and Its Composition, 1919–1938* (New York, Nat. Bur. of Econ. Research, 1941), Vol. I, p. 147.

[a] All rates were computed exponentially by comparing the corresponding magnitudes at the beginning and end of each period.

[b] Each year represents the mid-point of a decade. For instance, 1884 indicates the average magnitude for the decade 1879–1888; 1924, the period 1919–1928; and so on.

possibly also of the per capita income, though the performance of both rates in the twenties appears to have been extremely encouraging. Not much can be said about the period after 1929, because real output during the thirties had certainly little to do with productive powers. Also, there has been so much controversy about the measurement of real income during the present war years that it is better to postpone judgment. Estimates obtained from the U. S. Commerce Department show that, in the thirteen years 1929-42, total and per capita real income increased at an average rate of 3.4 and 2.6 per cent, respec-

capita *real* income was growing indicates that the price level rose at an average rate of 1.7 per cent.

Since the burden of the debt depends on the rate of growth of money income, a secular rise in prices will lighten the burden. In this paper it was agreed, however, to maintain a constant price level.

tively. Finally, there are estimates by the National Industrial Conference Board going back to 1799; these are also given in Appendix B.

The rate at which real output can be expected to grow in the future is a question about which a present-day economist has amazingly little to say. The problem of making full use of available productive capacity (except for the last few years when the war offered a solution) has been so challenging that not much attention has been devoted to the problem of long-run expansion. Indeed, one hesitates to talk about the expansion of productive powers when unemployment still looms as the most pressing post-war problem.

In general it appears very unlikely that national income, or any economic series for that matter, can grow indefinitely at some constant percentage rate.[39] The rate of growth achieved in the United States in the period 1879-1928 was due to technological improvements, growth of the labor force, and the discovery of new resources. Whether much reliance can be placed on resources still to be discovered is hard to say. It is true, however, that improved technological methods find new applications for known resources and thus may have the same effect as an actual discovery of new ones. The rate of growth of the population has been slackening ever since about 1850, and the various estimates of future population growth predict a practically stationary if not declining population by 1980. Under these conditions, a 3 per cent rate of growth of real income may be too much to hope for, but a 2 per cent rate for the next 50 or even 100 years can probably be well defended.

We have to recognize that the main, and later on the only, propelling force in the economy will be technological improvements which should result in an ever-rising productivity per manhour. Only technological improvements can offset the diminishing productivity of investment which would be caused by the insufficient growth of the labor force and of natural resources. Whether new inventions will be forthcoming in sufficient numbers and whether they will be applied fast enough is hard to tell; one often gets the impression that the scientific age is just beginning, and that once monetary problems are solved, technological advance will proceed at a tremendous rate. On the other hand, one also cannot escape the impression that certain institutional developments, particularly the growth of huge corporations and monopolies, are not conducive to rapid technological change, and that the mere assurance of an adequate effective demand will not solve the whole problem. A thorough reform of the whole process of industrial research and particularly of the application of inventions may be needed as well.

[39] For instance, one cent invested at 2 per cent 1944 years ago would amount now to something like 768,000 billion dollars.

It thus follows that, if it is desired to have national income grow at a given rate, two conditions must be satisfied:

1. The total volume of monetary expenditures, public and private, must grow at the same rate;

2. Of the total volume of these expenditures, a sufficient amount should be directed toward increasing the efficiency of production, so as to allow the required volume of monetary expenditures to take place without a rise in prices.

Since government is absorbing a part of savings, it is of course desirable that its expenditures be productive. This productivity has nothing to do, however, with such questions as whether or not the assets constructed make a direct contribution to the federal treasury or are self-liquidating. As a matter of fact, the term "investment expenditures" may be misleading, because it is too closely associated with steel and concrete.[40] If healthier people are more productive, expenditures on public health satisfy these requirements. The same holds true for expenditures on education, research, flood control, resource development and so on. Finally, if institutional forces prevent the government from spending money on anything but leaf-raking, it should still absorb the savings unused by private enterprise and spend them on leaf-raking, relying on private investment to raise the efficiency of production, rather than do nothing at all and thus create a shortage of monetary expenditures and unemployment.[41] Of course, national income would be able to advance at a higher rate if governmental expenditures were productive in our sense. In 1940 total private and public expenditures on industrial and scientific research in the United States were less than 500 million dollars. What would be the result if this amount were doubled, tripled or multiplied ten times? Indeed, large-scale governmental participation in industrial and scientific research could become one of the major propelling forces in the economy.[42]

[40] A substantial part of efficiency-raising expenditures is usually treated as current costs, and does not appear under the heading of capital formation or investment.

[41] It is an interesting question whether private investment would be able to take place at all in an economy characterized by a chronic shortage of monetary expenditures.

[42] Expenditures on industrial research made by private business in 1940 amounted to about 300 million dollars. To this should be added some 50 millions spent by universities; the latter figure includes their expenditures on research in social sciences as well. The figures for federal expenditures on scientific and industrial research in 1940 are not available; in 1938, they amounted to some 52 millions, the largest share going to the Department of Agriculture. See U. S. National Resources Committee, *Research—A National Resource, Vol. I—Relation of the Federal Government to Research* (Washington, 1938), U. S. National Resources Planning Board, *Research—A National Resource, Vol. II—Industrial Research* (Washington, 1941).

Since the beginning of the war, federal expenditures on research, particularly in the fields connected with the war effort, have shown a marked increase. A bill recently introduced by Senator Kilgore would authorize an annual appropriation of 250 millions on

It is possible, or even likely, that, in spite of all these efforts, national income will grow at a *decreasing* percentage rate. Several possibilities should now be examined:

(a) The fall in the rate of growth is accompanied, or rather caused, by a declining propensity to save. The public prefers to consume a greater share of its income today; therefore, a smaller percentage is invested, and income cannot grow as fast as it otherwise would. If the decline in the propensity to save and therefore in α is proportional to that in r, the burden of the debt $\frac{\alpha}{r} i$ remains unchanged. If, however, r suffers a greater proportional decline than α, we have the next case (b).

(b) r declines while the propensity to save and α remain constant, or at least do not decline as fast (proportionally) as r. The result is a genuine diminishing productivity of investment: further investments of the same percentage of national income result in smaller and smaller percentage increases in income. Under these conditions, whether the investment be made by private enterprise or by the government, it is impossible to pay a constant percentage return on the investment without increasing indefinitely the relative share of the national income going to property owners. If such a course is regarded as impossible or undesirable, the rate of return on the amounts invested must go down as well. This would mean in the case under discussion here that the interest rate on bonds must be continuously reduced.[43]

All of this discussion, with the exception of the case (a) just considered, was based on the assumption that over a period of time α remained constant. It will be worth while to examine the not improbable case when α increases, *i.e.*, when the government borrows an increasing percentage of the national income. There are again several possibilities:

(c) α remains a constant fraction of the propensity to save, but the propensity to save itself rises. In other words, a larger percentage of national income is invested. If so, the rate of growth may also increase and thus leave the burden of the debt, $\frac{\alpha}{r} i$, unchanged. If, on the other hand, r does not rise—or at least does not rise as fast (proportionally) as α—the result is diminishing productivity of investment already discussed under (b).

(d) The propensity to save remains constant, but α increases. In other words, a larger fraction of total savings is absorbed by the gov-

subsidies to various research organizations and on direct research by the federal government. The amount is rather small, but may prove to be a good beginning.

[43] It is very amusing that those who appear most worried about the burden of the debt are usually least willing to advocate a lower interest rate on the debt!

ernment and a smaller one by private business. As the propensity to save remains constant, there is no reason to expect an increase in r. Therefore, the ratio $\frac{\alpha}{r} - i$ and, hence, the burden of the debt will increase.

On the face of it, such a development appears quite unfavorable, since it was agreed to regard the debt burden as an evil which should be minimized. It is presumably an evil because a part of the national income has to be taken from the public and given to the bondholders. But if interest charges on the public debt are treated in this manner, a question arises why other forms of property income should be treated differently. After all, in peacetime society has a choice (at least in theory) of having its investment undertaken by the government or by private business. In the first case, a fixed return is given to the bondholders, and presumably neither the interest nor the principal is subject to default. In the second case, society promises the investors nothing, but allows them, subject to certain rules, to get whatever they can. Which method will result in a more rapidly rising national income is a question on which many opinions have been expressed but few, if any, studies ever undertaken. Nor has any serious attempt been made (at least to my knowledge) to analyze the possible changes in the magnitude of property income produced by a replacement of private investment by government investment. Too often has it been implicitly assumed that interest on government bonds is necessarily a net *addition* to other property income, rather than a *substitution* for other forms of property income; or, in other words, that investment by government, rather than by private business, must increase the magnitude of income going to property owners. Since this may or may not be true, there is no ground as yet for asserting that government investment raises the "burden" of the total, public and private, debt, that it increases the concentration of wealth and income, that it accelerates the growth of the *rentier* class, or that it raises the community's propensity to save—thus creating new difficulties all of which would be absent if the investment were done solely by private business.

There is also the question whether the transfer of income to property owners by means of taxation is more or less "painful" to the public or disturbing to the economy than a transfer of an equal amount by means of higher prices or lower wages.

The whole problem needs further study.

It is hoped that this paper has shown that the problem of the debt burden is essentially a problem of achieving a growing national income. A rising income is of course desired on general grounds, but in addition to its many other advantages it also solves the most important

aspects of the problem of the debt. The faster income grows, the lighter will be the burden of the debt.

In order to have a growing income there must be, first of all, a rising volume of monetary expenditures. Secondly, there must be an actual growth in productive powers in order to allow the increasing stream of expenditures to take place without a rise in prices.

When post-war fiscal policy is discussed, the public debt and its burden loom in the eyes of many economists and laymen as the greatest obstacle to all good things on earth. The remedy suggested is always the reduction of the absolute size of the debt or at least the prevention of its further growth. If all the people and organizations who work and study, write articles and make speeches, worry and spend sleepless nights—all because of fear of the debt—could forget about it for a while and spend even half their efforts trying to find ways of achieving a growing national income, their contribution to the benefit and welfare of humanity—and to the solution of the debt problem—would be immeasurable.

MATHEMATICAL APPENDIX

Y = national income; D = public debt; $U = Di$ = interest charges on the debt; $T = Y + U$ = taxable income; $\frac{U}{T}$ = tax rate; $Y' = Y\left(1 - \frac{U}{T}\right)$ = net income of the non-bondholders after the payment of taxes; a = national income at the beginning of the "experiment"; α = percentage of national income borrowed by the government; i = interest rate paid on the debt; b = absolute annual rate of growth of national income (in Case 2); r = percentage annual rate of growth of national income (in Cases 3 and 4); t = time (in years).

Case 1.

$$Y = a;$$

$$D = D_0 + \alpha a t;$$

$$\frac{D}{Y} = \frac{D_0}{a} + \alpha t; \tag{1}$$

$$\lim_{t \to \infty} \frac{D}{Y} = \infty; \tag{2}$$

$$\frac{U}{T} = \frac{Di}{Y + Di} = \frac{1}{\frac{Y}{Di} + 1};$$

$$\lim_{t\to\infty} \frac{U}{T} = 1 = 100 \text{ per cent}; \tag{3}$$

$$\lim_{t\to\infty} Y' = Y\left(1 - \lim_{t\to\infty} \frac{U}{T}\right) = 0. \tag{4}$$

Case 2.

$$Y = a + bt;$$

$$D = D_0 + \alpha \int_0^t (a + bt)dt$$

$$= D_0 + \alpha t\left(a + \frac{b}{2}t\right);$$

$$\frac{D}{Y} = \frac{D_0 + \alpha t\left(a + \frac{b}{2}t\right)}{a + bt}; \tag{5}$$

$$\lim_{t\to\infty} \frac{D}{Y} = \infty; \tag{6}$$

$$\lim_{t\to\infty} \frac{U}{T} = 1 = 100 \text{ per cent}; \tag{7}$$

$$Y' = Y\left(1 - \frac{U}{T}\right) = \frac{Y^2}{Y + U};$$

$$\lim_{t\to\infty} Y' = \frac{2b}{\alpha i}.$$

It can be readily shown from (5) that $\frac{D_1}{Y_1} < \frac{D_2}{Y_2}$ if $b_1 > b_2$, other parameters remaining the same. This also holds true for $\frac{U}{T}$.

Case 3.

$$Y = ae^{rt}$$

$$D = D_0 + \alpha a \int_0^t e^{rt}dt = D_0 + \frac{\alpha a}{r}(e^{rt} - 1);$$

$$\frac{D}{Y} = \frac{D_0}{ae^{rt}} + \frac{\alpha}{r}(1 - e^{-rt}); \tag{9}$$

$$\lim_{t\to\infty} \frac{D}{Y} = \frac{\alpha}{r}; \tag{10}$$

$$(11) \qquad \operatorname{Lim}_{t\to\infty} \frac{U}{T} = \frac{i}{\frac{r}{\alpha} + i} \cdot$$

Case 4. The "War Model"

Additional Symbols:

p = length of the "peace" period; α = percentage of national income borrowed during the "peace" period; w = length of the "war" period; β = percentage of national income borrowed during the "war" period; $\sigma = \frac{\alpha p + \beta w}{p + w}$ = the average percentage of national income borrowed.

Only the final results are given here; the derivations are available.[44]

$$(12) \qquad \text{Maximum} \quad \operatorname{Lim}_{t\to\infty} \frac{D}{Y} = \frac{\alpha + Ke^{rp}}{r};$$

$$(13) \qquad \text{Minimum} \quad \operatorname{Lim}_{t\to\infty} \frac{D}{Y} = \frac{\alpha + K}{r};$$

$$(14) \qquad \text{where} \quad K = \frac{(\beta - \alpha)(e^{rw} - 1)}{e^{(p+w)r} - 1};$$

$$(15) \qquad \text{Average} \quad \operatorname{Lim}_{t\to\infty} \frac{D}{Y} = \frac{\sigma}{r};$$

$$(16) \qquad \text{Maximum} \quad \operatorname{Lim}_{t\to\infty} \frac{U}{T} = \frac{(\alpha + Ke^{rp})i}{r + (\alpha + Ke^{rp})i};$$

$$(17) \qquad \text{Minimum} \quad \operatorname{Lim}_{t\to\infty} \frac{U}{T} = \frac{(\alpha + K)i}{r + (\alpha + K)i};$$

$$(18) \qquad \text{Average} \quad \operatorname{Lim}_{t\to\infty} \frac{U}{T} = \frac{i}{\frac{r}{\sigma} + i} \cdot$$

In expressions (15) and (18) a simple arithmetic average is used.

The expression (18) is actually an approximation of the true value of Aver. $\operatorname{Lim}_{t\to\infty} \frac{U}{T}$. It can be shown that the difference between them is likely to be very small and that (18) always overstates the true magnitude of Aver. $\operatorname{Lim}_{t\to\infty} \frac{U}{T}$.

[44] Please write the author, c/o Board of Governors of the Federal Reserve System, Washington, D.C.

APPENDIX B

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Table VI is presented here merely as an illustration: the data are not sufficiently comparable and are too fragmentary to warrant a more serious use. Definitions and accuracy of measurement vary from country to country. In addition, some figures were deflated by a cost-of-living index, while an index of wholesale prices had to be used for others. The relatively low rates of growth

TABLE VI.—PERCENTAGE RATES OF GROWTH OF REAL INCOME, TOTAL AND PER CAPITA, IN VARIOUS COUNTRIES[a]

Country	Period	Rate of Increase of Total Real Income	Rate of Increase of Per Capita Real Income	Percentage of Income Invested (Current Prices)
Australia	1901–03–1928–29	3.0	1.1	
	1921–22–1928–29	4.6	3.6	
	1901–03–1937–38	2.6	1.0	
	1921–22–1937–38	2.8	1.9	8.8[b]
Canada	1919 –1929	3.6	1.7	
	1919 –1940	2.5	1.0	
Germany	1891 –1913	1.8	0.5	18.0
Great Britain	1880 –1891–95	3.4	2.6	
	1891–95–1913	1.5	0.6	11.1
Hungary	1925–26–1936–37	1.9	1.2	4.8
Japan	1919 –1936	3.9	2.5	
New Zealand	1926 –1940	3.0	2.0	
Sweden	1913 –1930	2.3	1.8	11.2
	1922 –1930	2.9	2.6	10.5
United States				
N.I.C.B.[c]	1799 –1859	3.6	0.6	
	1879 –1929	3.2	1.4	
	1799 –1929	3.3	0.8	
Kuznets[d]	1884 –1924	3.3	1.5	13.3
	1919 –1929	4.2	2.7	10.8
U.S. Dept of Commerce	1929 –1942	3.4	2.6	6.3

[a] All rates were computed exponentially by comparing the corresponding magnitudes at the beginning and at the end of each period.

[b] Average for years 1928–29 through 1937–38.

[c] National Industrial Conference Board.

[d] See Table V, p. 818.

obtained for Germany may be due to the fact that a wholesale price index was used as a deflator.[45]

Sources of the figures for each country and the deflator used to get real income are given below.

Australia—Income, deflated by an index of prices of consumption and investment goods: Colin Clark and J. G. Crawford, *The National Income of Australia* (Sydney and London, 1938), p. 65. Investment: Clark, *The Conditions of Economic Progress* (London, 1940), p. 406.

Canada—Income, deflated by index of cost-of-living: *Monthly Review of Business Statistics,* April, 1943. Population: *Canadian Yearbooks,* 1940 and 1942.

Germany—Income figures, deflated by wholesale price index: *Das Deutsche Volkseinkommen vor und nach dem Kriege,* bearbeitet im Statischen Reichsamt, 1931, p. 68. Savings, as a percentage of income: Leon Goldenberg, *Income and Savings in France 1871-1914* (unpublished), p. 139.

Great Britain—Income, deflated by cost-of-living index: A. L. Bowley, *Wages and Income Since 1860* (Cambridge, 1937), p. 94. Savings as percentage of income: Leon Goldenberg, *Income and Savings in France 1871-1914* (unpublished), p. 145.

Hungary—All figures: Matthias Matolcsy and Stephen Varga, *The National Income of Hungary* (London, 1938), pp. 68 *ff.* The deflator used was a comprehensive price index.

Japan—Income, in current prices: *Mitsubishi Economic Research Bureau Monthly Circular,* April, 1937, p. 12. Deflated by index of wholesale prices: the *Federal Reserve Bulletin.* Population: *Japan Yearbook,* 1937.

New Zealand—Income, in fiscal years: *New Zealand Official Yearbook,* 1937, 1938, and 1943, interpolated to calendar years and deflated by index of retail prices from same source. Population: *Official Yearbook.*

Sweden—All figures: E. Lindahl, E. Dahlgren, and K. Koch, *National Income of Sweden 1861-1930* (London, 1937). The deflator was a cost-of-living index.

United States—N.I.C.B. figures: Income, deflated by an index of the general price level: Robert F. Martin, *National Income in the United States, 1799-1938,* National Industrial Conference Board, Inc. (New York, 1939), p. 6.

Kuznet's figures: Income deflated by a comprehensive price index. See Table V, page 818.

U. S. Department of Commerce figures: Income, deflated by comprehensive price index: National Income Unit of the Bureau of Foreign and Domestic Commerce.

[45] The period 1891-1913 was one of rising prices, and wholesale prices were rising faster than the cost of living. For instance, during this period the wholesale price index in England rose by 26.6 per cent, while the rise in the cost-of-living index was only 17 per cent. It is very likely that if the national income in Germany were deflated by a cost-of-living index, it would show a higher rate of growth than given in the table. Such an index, however, was not available.